

Supplementary Document

This document is divided into three parts. The first restates the relevant metalog theory introduced by Keelin [2016] for completeness. The second presents methodological and implementation details specific to our work. The third provides additional results. The source code and other supplementary materials are available at <https://www.cg.tuwien.ac.at/MM>.

S1 Background on Metalogs

This section restates key definitions and methods of the metalog framework introduced by Keelin [2016].

S1.1 Probability Density Function

Here, we present the probability density function (PDF) of the unbounded metalog distribution, defined on the real line. It is derived by differentiating the quantile function $M_t^u(y)$ with respect to the cumulative probability y and can be expressed in recursive form:

$$m_t^u(y) = \begin{cases} \frac{y(1-y)}{a_2}, & t = 2, \\ \left[\frac{a_2}{y(1-y)} + a_3 \left(\frac{y-0.5}{y(1-y)} + \ln \frac{y}{1-y} \right) \right]^{-1}, & t = 3, \\ \left[\frac{a_2}{y(1-y)} + a_3 \left(\frac{y-0.5}{y(1-y)} + \ln \frac{y}{1-y} \right) + a_4 \right]^{-1}, & t = 4, \\ \left[\frac{1}{m_{t-1}(y)} + a_t \frac{t-1}{2} (y-0.5)^{\frac{t-3}{2}} \right]^{-1}, & t \geq 5, t \text{ odd}, \\ \left[\frac{1}{m_{t-1}(y)} + a_t \left(\frac{(y-0.5)^{\frac{t}{2}-1}}{y(1-y)} + \left(\frac{t}{2} - 1 \right) (y-0.5)^{\frac{t}{2}-2} \ln \frac{y}{1-y} \right) \right]^{-1}, & t \geq 6, t \text{ even}, \end{cases} \quad (\text{S.1})$$

where $a_1, a_2, \dots$ denote the metalog coefficients and t their number. The PDF of the bounded metalog is derived from the unbounded case via a monotone transformation that restricts the support to a finite interval; the corresponding formula is provided in Eq. (6).

S1.2 Parameter Estimation

This section summarizes the metalog parameter estimation procedure originally developed by Keelin [2016] for completeness.

Because the metalog quantile function in Eq. (3) is linear in the coefficient vector $\mathbf{a} = (a_1, \dots, a_t)^\top$, parameter estimation reduces to a linear regression problem. Specifically, we consider a set of m distinct cumulative distribution function (CDF) data points $(\mathbf{x}, \mathbf{y}) = \{(x_i, y_i)\}_{i=1}^m$, where $0 < y_i < 1$. The coefficients $\mathbf{a}$ satisfy the system

of linear equations

$$\begin{aligned} x_1 &= a_1 + a_2 \ln \frac{y_1}{1-y_1} + a_3 (y_1 - 0.5) \ln \frac{y_1}{1-y_1} + \dots, \\ &\vdots \\ x_m &= a_1 + a_2 \ln \frac{y_m}{1-y_m} + a_3 (y_m - 0.5) \ln \frac{y_m}{1-y_m} + \dots \end{aligned}$$

In matrix notation, this relationship can be written compactly as $\mathbf{x} = \mathbf{Y}\mathbf{a}$, where $\mathbf{x} \in \mathbb{R}^m$ and $\mathbf{a} \in \mathbb{R}^t$ are column vectors, and $\mathbf{Y} \in \mathbb{R}^{m \times t}$ is given by:

$$\mathbf{Y} = \begin{bmatrix} 1 & \ln \frac{y_1}{1-y_1} & (y_1 - 0.5) \ln \frac{y_1}{1-y_1} & (y_1 - 0.5) & \dots \\ & & \vdots & & \\ 1 & \ln \frac{y_m}{1-y_m} & (y_m - 0.5) \ln \frac{y_m}{1-y_m} & (y_m - 0.5) & \dots \end{bmatrix}.$$

When $m = t$, and all y_i are distinct, the matrix $\mathbf{Y}$ is square and invertible, yielding the unique solution $\mathbf{a} = \mathbf{Y}^{-1}\mathbf{x}$. For the more general case $m \geq t$, provided that $\mathbf{Y}$ has full column rank t , the coefficients are estimated via the linear least-squares (LLSQ) solution,

$$\mathbf{a} = (\mathbf{Y}^\top \mathbf{Y})^{-1} \mathbf{Y}^\top \mathbf{x}. \quad (\text{S.2})$$

Importantly, in either case the linear system that needs to be solved is always of dimension $t \times t$, independent of the number of CDF data points m . Under the assumption of additive Gaussian noise in $\mathbf{x}$, this LLSQ estimator coincides with the maximum likelihood estimator [Keelin 2016].

S2 Methodological and Implementation Details

S2.1 Hybrid Newton–Raphson Solver

When working with metalog PDFs, $\hat{m}_t(x)$ is not available in closed form, but must be evaluated numerically by solving the quantile function M_t for y and then evaluating $m_t(y)$ as introduced in §3.

To this end, we introduce a *hybrid Newton–Raphson method* with bisection safeguards. Given x , we seek $y \in (0, 1)$ such that $M_t(y) = x$, where the standard Newton–Raphson update is $y_{i+1} = y_i - (M_t(y_i) - x) / (dM_t/dy)|_{y_i}$. For metalogs, $dM_t/dy = 1/m_t(y)$ is directly available from the metalog PDF. The Newton–Raphson update then simplifies to

$$y_{i+1} = y_i - (M_t(y_i) - x) m_t(y_i).$$

To ensure robustness, we accept an update only if the next estimate y_{i+1} remains within a bracketing interval $[l, u]$ (initially $(0, 1)$) and provides sufficient reduction in $M_t(y_{i+1}) - x$. Otherwise, we take a bisection step in the bracketing interval: $y_{i+1} = (u + l)/2$. Finally we update either l or u to y_{i+1} , such that $[l, u]$ always contains the root. We use this hybrid approach throughout our implementation, combining the quadratic convergence of Newton–Raphson with the reliability of bisection to guarantee convergence. Finally, this approach extends directly to multivariate metalogs by solving first along the marginal and subsequently on the conditionals.

S2.2 Expectation–Maximization for Metalog Mixtures

We summarize the expectation–maximization (EM) procedure for fitting metalog mixtures. As discussed in §S1.2, under a Gaussian noise assumption, the metalog coefficient estimator, Eq. (S.2), coincides with the maximum likelihood estimator [Keelin 2016] (up

to regularization), which allows metalog fitting to be incorporated into the standard EM framework [Bishop and Bishop 2024].

Initialization. The mixture coefficients are initialized uniformly as $\pi_k = 1/K, \dots, 1/K$, where K is the number of components, with each $\pi_k \in [0, 1]$ and $\sum_k^K \pi_k = 1$. Component-specific metalog coefficient vectors $\mathbf{a}_k$ are initialized as follows:

- (1) Construct a $N \times K$ weight matrix w_{nk} in cumulative probability space, where N is the number of data points, such that $\sum_n^N w_{nk} = 1$ for each component k . To define the weights, data points are first assigned empirical CDF ranks $r_n \in (0, 1)$ based on the sorted data. Each component k is associated with a center c_k uniformly spaced in $[0, 1]$, and weights w_{nk} are obtained by evaluating a normalized Gaussian kernel over $(r_n - c_k)$, assigning each component a localized $1/K$ -sized slice of the ordered data.
- (2) Build the design matrix $\mathbf{Y}$ (§S1.2) for a predefined sequence of cumulative probabilities $\{y_j\}$, evenly spaced in $(0, 1)$.
- (3) Using the normalized weights w_{nk} , we sample the empirical CDF at the selected probabilities $\{y_j\}$. To do so, the data $\{x_n\}$ are first sorted to obtain $x_1 \leq \dots \leq x_N$, and the weights w_{nk} are reordered accordingly. For each component k , the weights are normalized and accumulated to form a cumulative sum $P_k(x_n) = \sum_{i=1}^n w_{ik}$. For each target probability y_j , the index m satisfying $P_k(x_{m-1}) < y_j \leq P_k(x_m)$ is identified, and the quantile value q_{kj} is obtained by linear interpolation between x_{m-1} and x_m .
- (4) For each component k , solve Eq. (S.2) to obtain the initial coefficient vector $\mathbf{a}_k$.

Expectation Step (E-Step). Given the current parameters $\{\pi_k\}$ and $\{\mathbf{a}_k\}$, posterior responsibilities $\gamma_k(x_n)$ are computed according to Eq. (15). Evaluating these requires computing the metalog density as a function of $\hat{m}_i^u(x) = m_i^u((M_i^u)^{-1}(x))$. Since the metalog CDF has no closed-form expression, y is obtained numerically using the hybrid Newton–Raphson method described in §S2.1.

Maximization Step (M-Step). The mixture coefficients are updated in the standard manner as

$$\pi_k = \frac{1}{N} \sum_n \gamma_k(x_n).$$

To reestimate the metalog coefficients, per-component normalized data weights are defined as

$$w_{nk} = \frac{\gamma_k(x_n)}{\sum_i^N \gamma_k(x_i)}. \quad (\text{S.3})$$

Using these weights, weighted empirical quantiles q_{kj} are recomputed at the probabilities $\{y_j\}$. Each coefficient vector $\mathbf{a}_k$ is then reestimated by solving the LLSQ system in Eq. (S.2). This step generalizes the moment-based updates used in Gaussian mixture models to quantile-based parameter estimation.

At each iteration l , the data log-likelihood ℓ_l , Eq. (14), is evaluated, and the algorithm terminates when $|\ell_l - \ell_{l-1}| < \epsilon$. In summary, metalog mixture models can be fitted using a standard EM procedure, with quantile-based metalog estimation replacing moment-based updates. This formulation enables efficient maximum likelihood estimation for mixture metalogs.

S2.3 4D Metalog Material Fitting

With the LLSQ fitting for individual metalog distributions and the EM algorithm for fitting mixtures in place, we can now tackle the problem of fitting metalog materials to reference bidirectional reflectance distribution function (BRDF) data.

Given an analytic or data-driven reference BRDF f_r as input, we first choose a set of incident directions $\{\omega_i\}$ (on the hemispherical grid) and for each grid point perform the following steps:

- (1) Generate a set of samples $\{\omega_o, f_r(\omega_i, \omega_o)\}$, and estimate the directional-hemispherical reflectance $\rho_{\text{dh}}(\omega_i)$ by Monte Carlo (MC) integration.
- (2) Map the sampled ω_o to marginal θ_o and conditional ϕ_o axes of the 2D metalog structure.
- (3) Fit the marginal distribution of the metalog mixture to the samples $\{\theta_o\}$ (using the EM algorithm described above).
- (4) For a set of marginal quantiles (defining the conditional slices), compute normalized Gaussian kernel weights for all samples based on their marginal distance (in θ_o) to each slice. Using these weights, we form a discrete empirical distribution over the associated conditional samples ϕ_o . For each slice, we draw a new set of samples from this discrete distribution and fit a conditional metalog mixture using EM.
- (5) Store the resulting (marginal–conditional) metalog mixture model and the associated directional-hemispherical reflectance for the current ω_i .

S2.4 Implementation Details and Optimizations

In this section, we outline further implementation details and optimizations used in our framework.

Isotropic Materials. For isotropic materials, we can optimize our representation by reducing the dimensionality of both the incident-direction grid and the directional-hemispherical reflectance texture by one. This follows from rotational invariance about the surface normal: for a fixed incident elevation, the distribution of outgoing directions is independent of the absolute azimuth. As a result, incident directions can be parameterized solely by elevation, enabling a more compact discretization without loss of generality.

Rusinkiewicz Parameterization. An additional optimization is encoding directions using the Rusinkiewicz halfway-vector parameterization rather than directly representing outgoing directions. In this formulation, directions are expressed relative to the half vector between the incident and outgoing directions. This choice is advantageous for many common materials whose scattering behavior is concentrated around the specular reflection direction. By measuring angular deviations with respect to the reflection lobe rather than absolute directions, the resulting function typically exhibits reduced angular variation, which in turn improves the efficiency and robustness of the fitting process. Importantly, this parameterization is orthogonal to the derivations presented so far. Enabling it requires only a change of coordinates: outgoing directions are mapped into halfway-vector space during fitting, and the inverse transformation is applied during sampling. No modifications to the underlying estimator or theoretical framework are required. Isotropic materials are well represented by reduced parameterizations (fewer grid nodes N_g ,

Table S1), whereas strongly anisotropic or highly specular materials benefit from the full four-dimensional metalog model (Table S2).

S2.4.1 Material Editing Examples. In this section, we briefly describe how to modify the appearance by adjusting the marginal metalog coefficients. In Fig. 19, both materials are isotropic, and we fit bivariate metalog slices ($N_c = 8$, $K = 1$). In both cases, the marginal axis represents $\cos \theta_o$. To make the rough analytic material more specular (top row in Fig. 19), we adjust the coefficients across all marginal metalog slices. Specifically, the goal is to obtain a highly right-skewed distribution that generates more samples near the surface normal ($\theta_o \approx 0$), such that most samples lie in the range $\cos \theta_o \in [0.9, 1.0]$. This specular behavior can be achieved simply by adjusting the first coefficient a_1 across all marginal metalogs, which corresponds to the median of the distribution. Conversely, generating a more diffuse-looking material would require evenly distributed $\cos \theta_o$ samples across the entire $[0, 1]$ range. In the second row of Fig. 19, we do this for the marginal metalogs that we fitted for the incoming directions $(\theta_i, \phi_i) \in \{(1.24, 0.0), (1.39, 0.0), (1.57, 0.0)\}$ to obtain a more diffuse (and therefore brighter) appearance for the grazing angles. We show similar editing examples in Fig. S6.

S3 Extended Results

In Table S1, we provide an overview of our results for fitting metalog materials to the isotropic materials in the RGL database. Similarly, Table S2 summarizes results for the anisotropic materials in the RGL database and for Mitsuba roughconductor materials, which are based on the Trowbridge–Reitz (TR) distribution. We show rendered images for these material fitting experiments in Figs. S1 to S3, including comparisons to the neural BRDF (NBRDF) approach of Sztrajman et al. [2021]. For the anisotropic TR materials, we compare image-space errors and render times (at equal samples) in Fig. S4: metalog materials render slightly slower than the analytic model when using multiple importance sampling (MIS), but faster with standard importance sampling. Figure S5 shows metalog material interpolation spatially across a surface with different grayscale textures as blending weights. Figure S6 shows further appearance editing examples. Finally, Fig. S7 presents additional examples of materials acquired via differentiable rendering from real-world images, while Fig. S8 shows our capture setup.

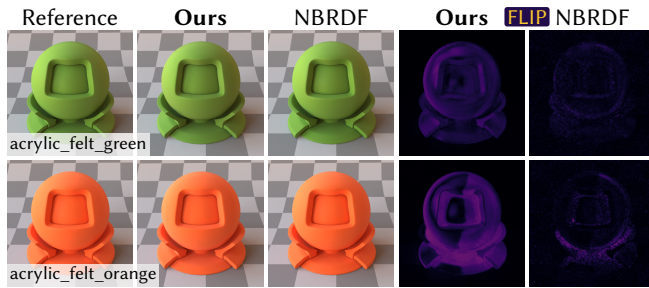


Fig. S1. Fitting isotropic RGL materials (1/6).



Fig. S1. Fitting isotropic RGL materials (2/6).

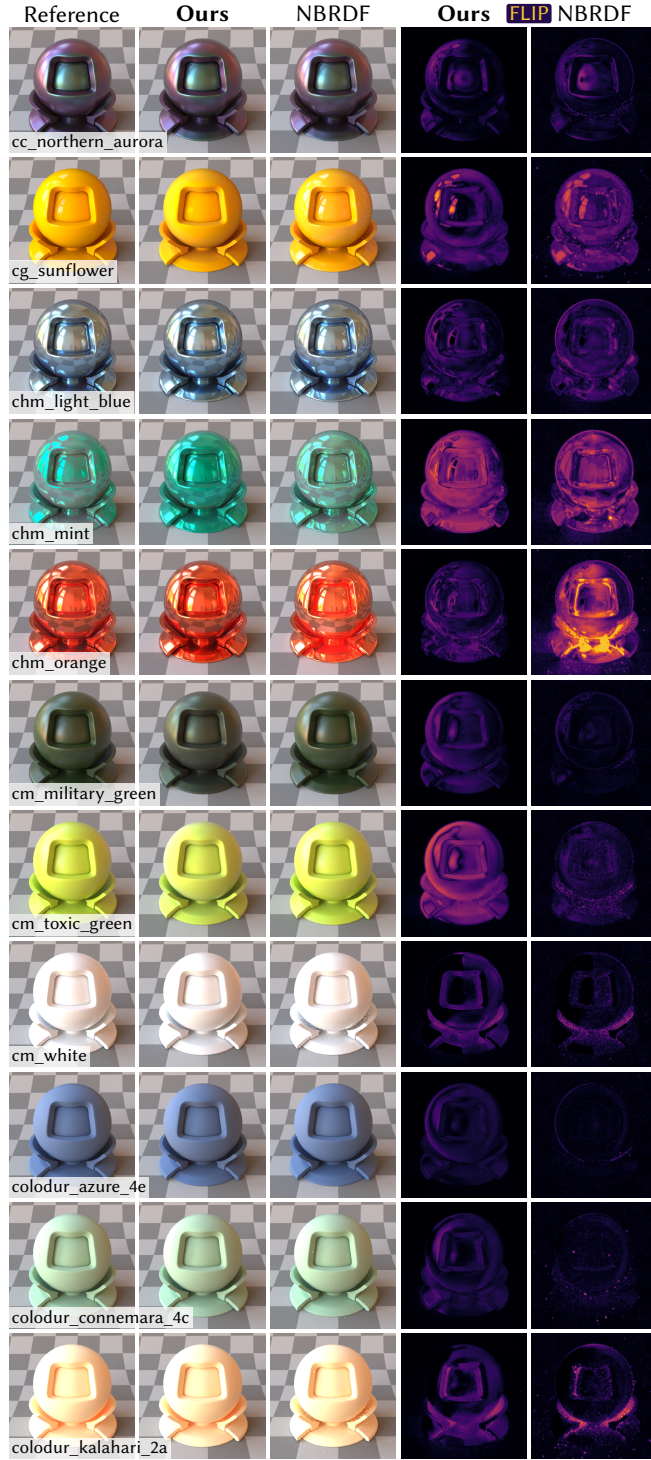


Fig. S1. Fitting isotropic RGL materials (3/6).



Fig. S1. Fitting isotropic RGL materials (4/6).



Fig. S1. Fitting isotropic RGL materials (5/6).

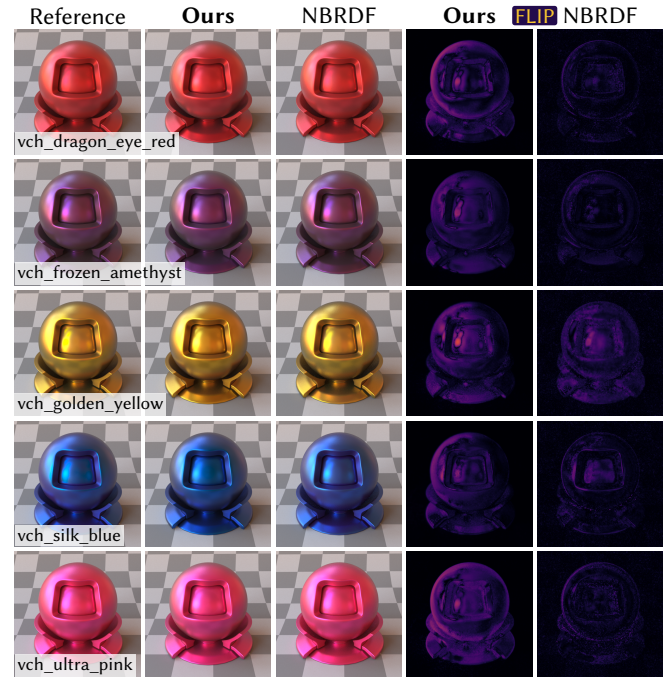


Fig. S1. Fitting isotropic RGL materials (6/6).

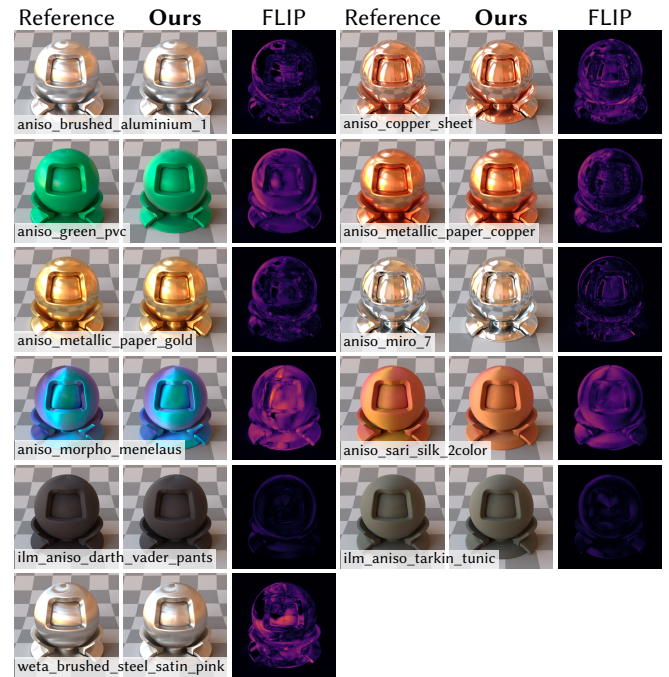


Fig. S2. Fitting anisotropic RGL materials.

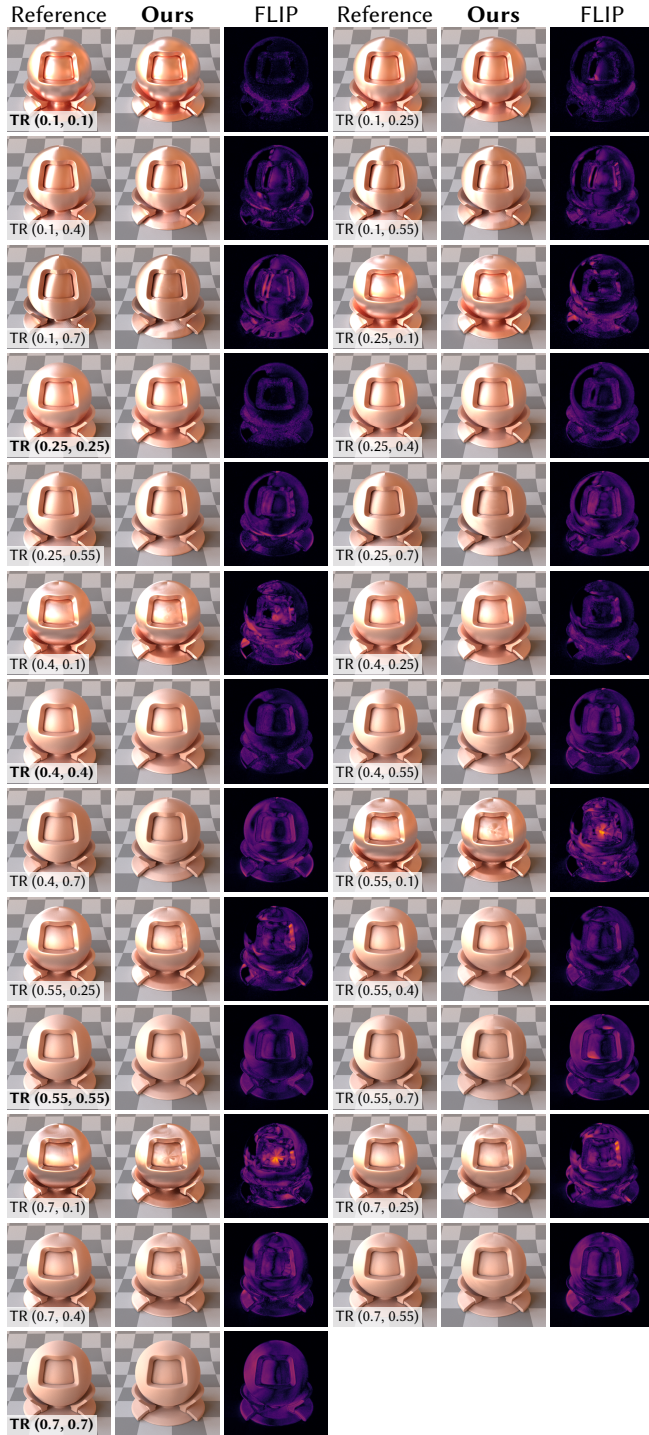


Fig. S3. Fitting Mitsuba’s TR-based roughconductor materials.

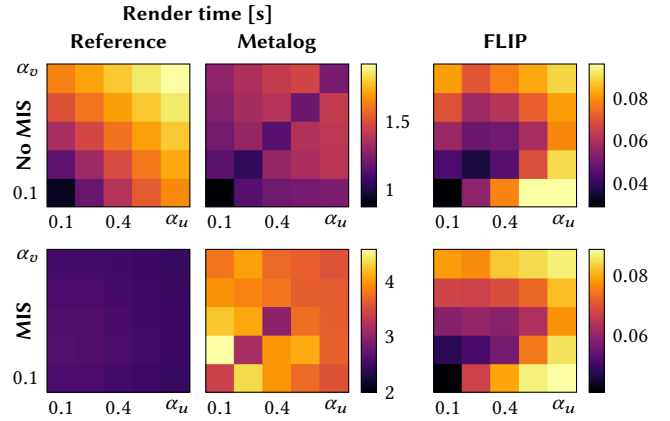


Fig. S4. Render times and FLIP errors for the materials in Fig. S3 (top: standard importance sampling, bottom: MIS).

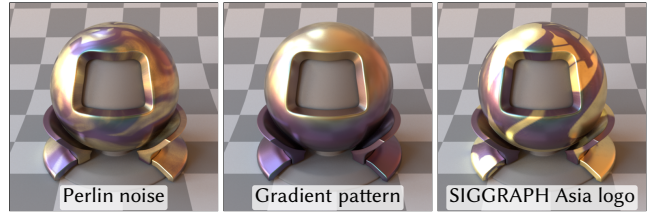
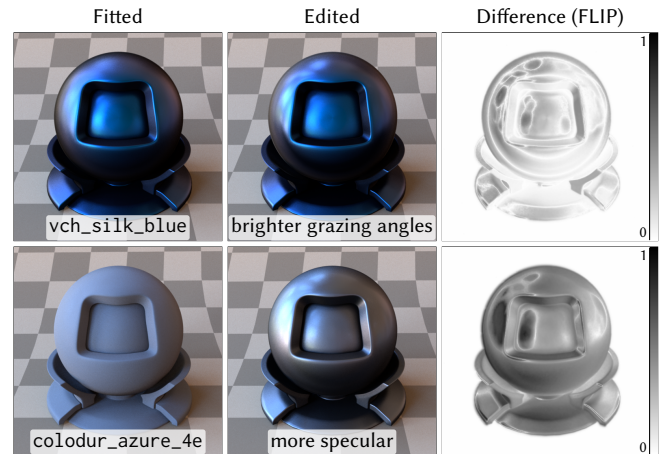
Fig. S5. Metalog interpolation naturally enables spatially varying bidirectional reflectance distribution functions (SVBRDFs): here we interpolate between RGL’s cc_northern_aurora and a TR material (gold, $\alpha = 0.1$) using different textures.

Fig. S6. Editing: we first fit a material from the RGL database and subsequently edit the fitted material to obtain the desired appearance.

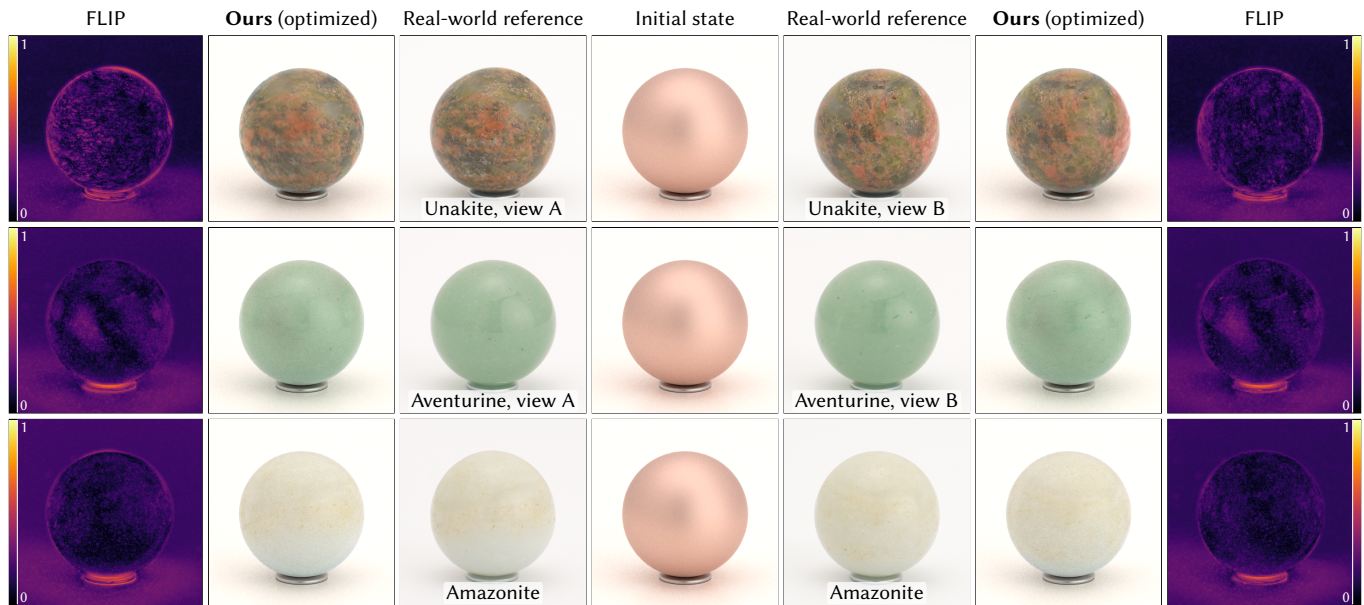


Fig. S7. Materials acquired via differentiable rendering from real-world images (using 8 views per sample in 45° rotation increments).



Fig. S8. Behind the scenes of our material acquisition system (Figs. 16, S7): We capture small material samples (marbles with approximately 16 mm diameter) on a motorized turntable using a single camera (Sony Alpha 6700 camera with a Sony E PZ 18–200 mm F3.5–6.3 OSS lens). The marbles are placed inside a custom-built light box made from white-laminated fiberboard, providing controlled and uniform illumination using two LED video lights (Neewer NL660-2.4G) equipped with softboxes and set to a correlated color temperature of 4400 K.

Table S1. Overview of fitting results (isotropic RGL materials). Columns: N_g number of grid nodes (incident direction), K_m number of marginal mixture components, N_m total number of marginal metalog coefficients (all components), S number of conditional slices, K_c total number of conditional mixture components (all slices), N_c total number of conditional coefficients (all slices and all components), ‘Mem.’ metalog memory footprint [KB] (single precision, 1 KB = 1024 bytes), $t_{\text{ours}}^{\text{MIS}}$ render time for metalog material using MIS [s], $t_{\text{ref}}^{\text{MIS}}$ render time for reference material using MIS [s], t_{ours} render time for metalog material (without MIS) [s], t_{ref} render time for reference material (without MIS) [s], t_{fit} fitting time [s], FLIP errors (ours vs. reference) with/without MIS.

Material (RGL database)	N_g	K_m	N_m	S	K_c	N_c	Mem.	$t_{\text{ours}}^{\text{MIS}}$	$t_{\text{ref}}^{\text{MIS}}$	t_{ours}	t_{ref}	t_{fit}	FLIP (MIS)	FLIP
acrylic_felt_green	10	1	7	6	6	47	2.62	2.60	2.64	1.15	1.08	132.08	5.1e-02	4.4e-02
acrylic_felt_orange	10	1	8	6	6	47	2.66	2.81	2.89	1.15	1.05	158.63	7.0e-02	5.6e-02
acrylic_felt_pink	10	1	6	6	6	46	2.54	2.67	2.71	1.15	1.04	134.04	6.3e-02	5.3e-02
acrylic_felt_purple	10	1	6	6	6	43	2.42	2.61	2.65	1.16	1.10	137.17	4.3e-02	3.9e-02
acrylic_felt_white	10	1	6	6	6	48	2.62	3.92	4.09	1.12	1.05	165.13	5.3e-02	3.5e-02
acrylic_felt_yellow	10	1	6	6	6	47	2.58	3.90	4.07	1.12	1.05	131.50	9.4e-02	6.3e-02
aurora_white	10	1	7	6	6	44	2.50	3.62	3.69	1.01	0.94	134.85	7.3e-02	8.6e-02
cardboard	10	1	8	6	6	44	2.54	3.30	3.29	1.11	1.02	126.37	5.5e-02	5.0e-02
cc_amber_citrine	8	1	12	6	6	34	1.84	2.17	2.55	0.57	0.84	185.75	1.2e-01	1.2e-01
cc_blue_agat	8	1	12	6	6	35	1.88	2.27	2.66	0.66	0.95	131.32	1.3e-01	1.2e-01
cc_green_malachite	8	1	11	6	6	40	2.00	2.44	2.48	0.95	0.91	131.64	8.6e-02	8.3e-02
cc_ibiza_sunset	8	2	12	6	6	43	2.16	2.40	2.36	0.84	0.74	200.07	4.3e-02	3.7e-02
cc_iris_purple_gem	8	2	12	6	6	41	2.09	2.32	2.30	0.76	0.74	193.54	4.1e-02	3.5e-02
cc_northern_aurora	8	2	12	6	6	44	2.19	2.40	2.36	0.86	0.79	192.35	4.0e-02	3.7e-02
cg_sunflower	10	1	7	6	6	48	2.66	3.63	3.61	1.10	0.90	145.48	7.6e-02	6.0e-02
chm_light_blue	8	1	7	6	6	44	2.00	2.29	2.42	0.85	0.96	196.80	5.4e-02	5.9e-02
chm_mint	8	1	6	6	6	39	1.81	2.40	2.60	0.93	1.17	156.53	1.5e-01	1.6e-01
chm_orange	8	1	11	6	6	40	2.00	2.46	2.57	0.81	0.72	188.15	5.8e-02	7.7e-02
cm_military_green	10	1	8	6	6	40	2.38	2.99	3.06	0.76	0.83	146.35	5.2e-02	5.3e-02
cm_toxic_green	10	1	7	6	6	46	2.58	3.50	3.48	1.10	0.95	127.25	1.3e-01	1.2e-01
cm_white	10	1	4	6	6	48	2.54	3.92	3.97	1.11	0.95	143.20	4.7e-02	3.1e-02
colodur_azure_4e	10	1	7	6	6	44	2.50	2.56	2.58	1.31	1.21	132.28	4.0e-02	3.8e-02
colodur_connemara_4c	10	1	8	6	6	48	2.70	2.54	2.57	1.28	1.18	127.85	4.6e-02	3.6e-02
colodur_kalahari_2a	10	1	8	6	6	37	2.27	3.27	3.49	1.28	1.17	135.35	6.0e-02	3.2e-02
colodur_napoli_4f	10	1	7	6	6	41	2.38	2.59	2.67	1.34	1.32	134.42	5.1e-02	4.9e-02
ilm_13_37_dark_green	8	2	12	6	6	44	2.19	2.40	2.39	0.82	0.73	197.67	3.8e-02	3.6e-02
ilm_13_37_matte	10	1	8	6	6	46	2.62	3.32	3.25	1.13	0.97	146.91	5.4e-02	4.4e-02
ilm_13_37_metallic	8	1	10	6	6	44	2.09	2.39	2.46	0.80	0.84	144.62	3.2e-02	2.4e-02
ilm_solo_m_68	8	1	11	6	6	28	1.62	1.89	2.09	0.52	0.62	169.67	8.7e-02	3.2e-02
ilm_solo_millennium_falcon	10	1	6	6	6	48	2.62	2.63	2.63	1.13	0.96	134.22	4.7e-02	3.3e-02
irid_flake_paint1	8	1	12	6	6	28	1.66	2.26	2.36	0.72	0.78	203.49	3.5e-02	3.6e-02
irid_flake_paint1_fine	8	1	12	6	6	31	1.75	2.80	2.87	0.71	0.73	175.99	3.4e-02	3.4e-02
irid_flake_paint2	8	1	12	6	6	41	2.06	2.36	2.44	0.79	0.83	144.44	4.0e-02	4.0e-02
laika_ceiling_paint_18_gray	10	1	6	6	6	44	2.46	3.22	3.22	1.10	1.04	133.86	2.8e-02	2.5e-02
leaf_maple	10	1	7	6	6	47	2.62	3.21	3.13	1.03	0.90	126.99	7.5e-02	7.5e-02
paper_blue	10	1	7	6	6	48	2.66	3.57	3.61	1.12	1.04	131.23	6.9e-02	6.1e-02
paper_green	10	1	7	6	6	47	2.62	3.40	3.41	1.12	1.03	129.45	8.7e-02	7.4e-02
paper_red	10	1	7	6	6	44	2.50	3.71	3.81	1.14	1.08	126.11	7.7e-02	7.4e-02
paper_white	10	1	6	6	6	47	2.58	4.72	5.06	1.07	1.03	158.42	4.7e-02	2.4e-02
paper_yellow	10	1	6	6	6	39	2.27	4.78	5.14	1.10	1.02	131.53	1.1e-01	8.3e-02
satin_blue	10	1	6	6	6	46	2.54	3.67	3.76	1.07	0.96	127.83	7.5e-02	8.3e-02
satin_gold	10	1	6	6	6	43	2.42	3.66	3.77	1.01	0.96	132.14	6.7e-02	7.5e-02
satin_purple	10	1	6	6	6	48	2.62	3.72	3.83	1.08	0.96	134.22	6.5e-02	6.7e-02
satin_rosaline	10	1	7	6	6	45	2.54	3.76	3.84	1.07	0.96	136.49	6.0e-02	6.3e-02
satin_white	10	1	6	6	6	43	2.42	3.76	3.81	1.00	0.93	137.31	4.3e-02	3.4e-02
spectralon	10	1	8	6	6	46	2.62	4.61	4.95	1.07	1.02	138.27	4.3e-02	2.1e-02
vch_dragon_eye_red	8	1	6	6	6	44	1.97	2.56	2.58	0.96	1.04	161.95	5.7e-02	5.3e-02
vch_frozen_amethyst	8	2	12	6	6	46	2.25	2.54	2.50	0.97	0.92	176.79	4.9e-02	4.8e-02
vch_golden_yellow	8	2	12	6	6	40	2.06	2.48	2.48	0.93	0.95	168.32	5.5e-02	5.4e-02
vch_silk_blue	8	2	10	6	6	42	2.06	2.50	2.49	0.94	0.99	186.81	4.7e-02	4.4e-02
vch_ultra_pink	8	2	8	6	6	48	2.19	3.46	3.47	0.96	0.99	176.53	6.8e-02	6.4e-02

Table S2. Overview of fitting results (anisotropic RGL and (α_u, α_v) -Trowbridge-Reitz materials). Columns: N_g number of grid nodes (incident direction), K_m number of marginal mixture components, N_m total number of marginal metalog coefficients (all components), S number of conditional slices, K_c total number of conditional mixture components (all slices), N_c total number of conditional coefficients (all slices and all components), ‘Mem.’ metalog memory footprint [KB] (single precision, 1 KB = 1024 bytes), $t_{\text{ours}}^{\text{MIS}}$ render time for metalog material using MIS [s], $t_{\text{ref}}^{\text{MIS}}$ render time for reference material using MIS [s], t_{ours} render time for metalog material (without MIS) [s], t_{ref} render time for reference material (without MIS) [s], t_{fit} fitting time [s], FLIP errors (ours vs. reference) with and without MIS.

Material (RGL database)	N_g	K_m	N_m	S	K_c	N_c	Mem.	$t_{\text{ours}}^{\text{MIS}}$	$t_{\text{ref}}^{\text{MIS}}$	t_{ours}	t_{ref}	t_{fit}	FLIP (MIS)	FLIP
aniso_brushed_aluminium_1	136	4	20	6	6	40	40.38	4.26	3.87	1.38	1.09	3473.18	5.4e-02	6.1e-02
aniso_copper_sheet	136	4	24	6	6	35	39.84	4.32	4.07	1.15	0.92	4411.43	6.7e-02	6.6e-02
aniso_green_pvc	136	2	12	6	6	59	45.16	2.81	3.29	0.63	1.18	1452.08	8.6e-02	9.4e-02
aniso_metallic_paper_copper	136	4	24	6	6	40	42.50	4.30	3.94	1.12	0.98	4034.02	5.7e-02	5.4e-02
aniso_metallic_paper_gold	136	4	24	6	6	28	36.12	4.39	4.07	1.17	0.97	3666.96	5.4e-02	5.0e-02
aniso_miro_7	136	3	12	6	6	40	35.59	3.98	3.44	0.80	0.74	3637.75	4.2e-02	1.9e-02
aniso_morpho_menelaus	136	4	16	8	8	84	63.75	3.72	4.14	0.76	1.57	3989.53	1.3e-01	1.2e-01
aniso_sari_silk_2color	136	4	24	8	8	79	65.34	3.68	4.02	0.78	1.30	3548.54	8.7e-02	9.0e-02
ilm_aniso_darth_vader_pants	136	1	5	6	6	44	32.94	2.67	3.40	0.60	1.32	703.30	2.2e-02	2.1e-02
ilm_aniso_tarkin_tunic	136	1	12	6	6	39	34.00	2.61	3.30	0.60	1.25	672.19	2.4e-02	2.3e-02
weta_brushed_steel_satin_pink	136	4	16	6	6	61	49.41	4.14	3.72	1.66	1.18	3109.19	8.3e-02	9.4e-02
Material (TR (α_u, α_v))	N_g	K_m	N_m	S	K_c	N_c	Mem.	$t_{\text{ours}}^{\text{MIS}}$	$t_{\text{ref}}^{\text{MIS}}$	t_{ours}	t_{ref}	t_{fit}	FLIP (MIS)	FLIP
TR (0.1, 0.1)	10	1	6	6	6	45	2.50	3.39	2.58	0.87	0.93	80.36	4.1e-02	2.9e-02
TR (0.1, 0.25)	170	2	12	6	6	41	44.49	4.57	2.64	1.14	1.15	2252.96	4.8e-02	4.4e-02
TR (0.1, 0.4)	170	2	6	6	6	34	35.86	4.23	2.62	1.21	1.35	2445.30	5.9e-02	5.7e-02
TR (0.1, 0.55)	170	2	6	6	6	37	37.85	3.96	2.59	1.25	1.50	2233.49	6.8e-02	7.0e-02
TR (0.1, 0.7)	170	2	12	6	6	34	39.84	3.79	2.55	1.28	1.64	2114.97	7.8e-02	8.2e-02
TR (0.25, 0.1)	170	2	10	6	6	48	47.81	4.33	2.57	1.14	1.18	2281.60	6.7e-02	5.5e-02
TR (0.25, 0.25)	10	1	6	6	6	42	2.38	3.14	2.62	1.04	1.32	78.97	5.1e-02	3.7e-02
TR (0.25, 0.4)	170	2	6	6	6	48	45.16	4.07	2.61	1.30	1.47	2024.23	6.0e-02	4.9e-02
TR (0.25, 0.55)	170	2	12	6	6	35	40.51	3.89	2.58	1.34	1.60	2276.06	6.7e-02	5.8e-02
TR (0.25, 0.7)	170	2	10	6	6	32	37.19	4.03	2.54	1.36	1.71	2406.77	7.6e-02	7.0e-02
TR (0.4, 0.1)	170	2	10	6	6	48	47.81	3.98	2.54	1.19	1.38	2164.27	7.9e-02	7.8e-02
TR (0.4, 0.25)	170	2	6	6	6	48	45.16	3.99	2.57	1.30	1.48	1987.58	5.6e-02	4.6e-02
TR (0.4, 0.4)	10	1	6	6	6	36	2.15	2.98	2.57	1.14	1.60	77.22	5.9e-02	5.0e-02
TR (0.4, 0.55)	170	2	6	6	6	48	45.16	3.78	2.54	1.38	1.70	1911.56	6.9e-02	6.2e-02
TR (0.4, 0.7)	170	2	10	6	6	32	37.19	3.70	2.51	1.42	1.80	2075.68	8.3e-02	7.7e-02
TR (0.55, 0.1)	170	2	8	6	6	48	46.48	3.71	2.49	1.21	1.54	2237.58	8.7e-02	9.6e-02
TR (0.55, 0.25)	170	2	10	6	6	48	47.81	4.09	2.52	1.35	1.62	2239.20	7.4e-02	6.9e-02
TR (0.55, 0.4)	170	2	12	6	6	41	44.49	3.74	2.52	1.37	1.71	1880.50	6.3e-02	5.9e-02
TR (0.55, 0.55)	10	1	6	6	6	37	2.19	3.64	2.50	1.19	1.79	82.71	7.3e-02	7.2e-02
TR (0.55, 0.7)	170	2	10	6	6	34	38.52	3.65	2.48	1.46	1.87	1849.09	8.4e-02	8.3e-02
TR (0.7, 0.1)	170	2	10	6	6	33	37.85	3.55	2.44	1.24	1.66	2076.13	8.9e-02	9.6e-02
TR (0.7, 0.25)	170	2	12	6	6	38	42.50	3.65	2.48	1.40	1.72	2400.43	8.5e-02	9.0e-02
TR (0.7, 0.4)	170	2	12	6	6	30	37.19	3.64	2.48	1.40	1.80	2022.05	7.7e-02	7.8e-02
TR (0.7, 0.55)	170	2	12	6	6	48	49.14	3.62	2.46	1.42	1.87	1828.25	8.2e-02	8.2e-02
TR (0.7, 0.7)	10	1	6	6	6	39	2.27	3.55	2.44	1.22	1.93	79.24	8.7e-02	8.9e-02